A Mathematical Model of Beauty for Sustainable Urban Design

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用C.亚历山大的客观美重新审视大数据时代的可持续城市设计
A Mathematical Model of Beauty for Sustainable Urban Design

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Content (four parts)

- Review of Alexander's life works as a whole
- Fractals or living structures emerged from big data
  - Head/tail breaks
  - Natural cities
- A mathematical model of beauty
  - Beauty for individual centers
  - Beauty for the whole
- Sustainable urban design (adaptation + differentiation)

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Part 1
Review of Alexander's life works as a whole

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A brief CV of Alexander

- Born in Vienna, Austria in 1936
- 1954-1956: math degree at Cambridge
- 1956-1958: architecture degree at Cambridge
- 1958-1964: PhD degree at Harvard
- 1967: founded the Center for Environmental Structure (design + build)
- 1983: his biography by Stephen Grabow
- 1996: fellow of the American Academy of Arts and Science
- Now live in West Sussex near London UK.

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On Alexander's work

- "...Five hundred years is a long time, and I don't expect many of the people I interview will be known in the year 2500. Christopher Alexander may be an exception."
- "Could very well be the most read architectural treatise of all time."
- " A single-handedly trying to destroy the trillion dollar construction industry"
- "Exceptionally influential among Americans who are thinking about the design of their environments"
- "Undeniably an extraordinary piece of literature"
- "A revelation"

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Modern architecture

- *Modern (and post-modern) architecture is based on coming up with a picture or image and then constructing it, usually out of components and modular parts.*
- This way of doing architecture is essentially governed by mechanistic worldview.

Mechanistic world view

- *The mechanistic idea of order can be traced to Descartes, about 1640.*
- *If you want to know how something works, you can find out by pretending that it is a machine.*
- *However...*
- *It was because of this kind of Cartesian thought that one was able to find out how things work in the modern sense.*

Two devastating results

- *The first was that the "I" went out of our world-picture. The picture of the world as a machine doesn't have an "I" in it. The "I", what it means to be a person, the inner experience of being a person, just isn't part of this picture.*
- *The picture of the world we have from physics, because it is built only out of mental machines, no longer has any definite feeling of value in it: value has become sidelined as a matter of opinion, not intrinsic to the nature of the world at all.*

Beauty is NOT just in the eye of the beholder

- The statement of good architecture is true or false rather than only a matter of opinion.
- Beauty exists in the deep structure.

A dead tree is more alive than a modern building

A Vision for Architecture as More Than the Sum of Its Parts

How Modernist Fundamentalism degrades the human and natural environment

Jay Walljasper

Michael W. Mehaffy & Nikos A. Salingaros

Posted November 11, 2013

Arts and Culture

Community Life

Slide 13

The Architect Has No Clothes
Why so much modern design looks harsh and feels inhospitable

Jay Walljasper

Michael Mehaffy & Nikos A. Salingaros

Posted October 15, 2011

Arts and Culture

Community Life

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Slide 14

Ulrich R. S. (1984), View through a window may influence recovery from surgery, *Science*, 224, 420-422.

Slide 15

Alexander's major works among many others

- Notes on the Synthesis of Form (1964)
- A City Is not a Tree (1965)

- A Pattern Language (1977)
- The Timeless Way of Building (1979)
- The Oregon Experiment (1975)

- The Nature of Order (Book 1, 2, 3 and 4) (2002-2005)

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Slide 16

Notes on the Synthesis of Form

- Unselfconscious
- Selfconscious

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Slide 17

What if 师傅 died?

A LIVING ARCHITECTURE

FRANK LLOYD WRIGHT AND TALIESIN ARCHITECTS

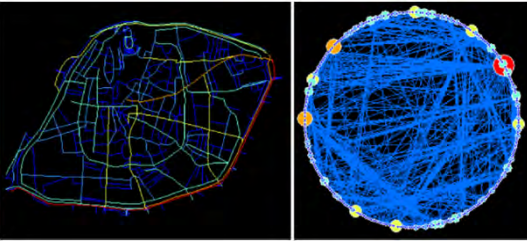JOHN RATTENBURY

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Slide 18
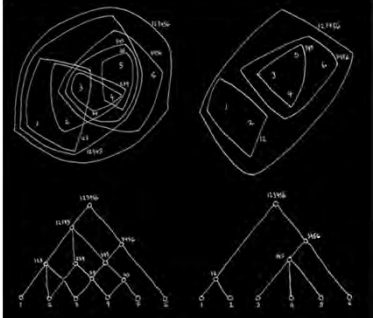
A city is not a tree

18

A city is a complex network

Jiang B. (2015), A city is a complex network, in: M. W. Mehaffy (editor, 2015), *Christopher Alexander A City is Not a Tree: 50th Anniversary Edition,* Sustasis Press: Portland, OR, 89-98.
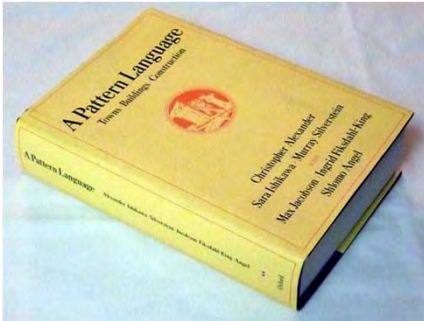
19

Discovery versus invention

- Christopher Alexander
- Benoit Mandelbrot
- Thomas Edison
- Steve Jobs

20
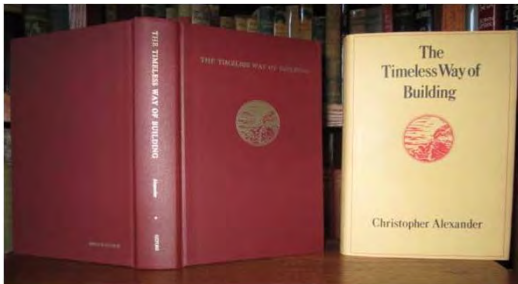
A Pattern Language (253 patterns)

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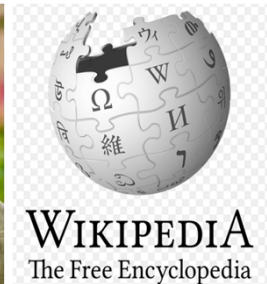## The Oregon Experiment

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The Timeless Way of Building

- The quality without a name

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Impact of pattern language

- Ward Cunningham

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The failure of pattern languages

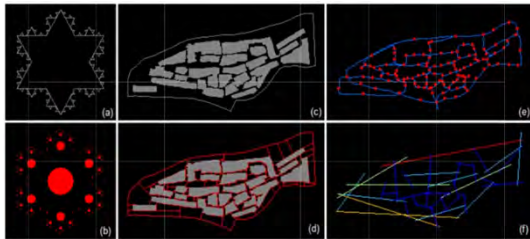## Which one is more alive or beautiful?
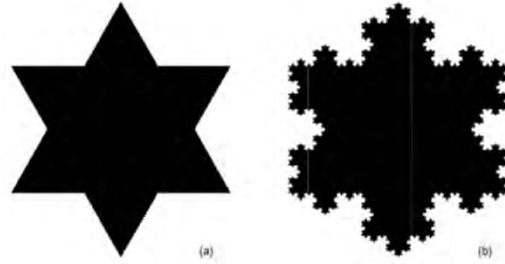
Again…

Wholeness, life and beauty (Maggie's circle)

Fifteen geometric properties

Jiang B. (2015), Wholeness as a hierarchical graph to capture the nature of space, *International Journal of Geographical Information Science*, 29(9), 1632–1648.
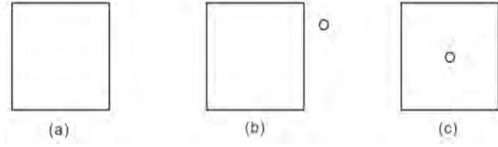
The Nature of Order

The major thesis of the book

- The order (or harmony or coherence) in nature and in what we build or make are essentially the same.

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A structure with or without a center

(a)   (b)   (c)

Jiang B. and Sui D. (2014), A new kind of beauty out of the underlying scaling of geographic space, *The Professional Geographer*, 66(4), 676–686.
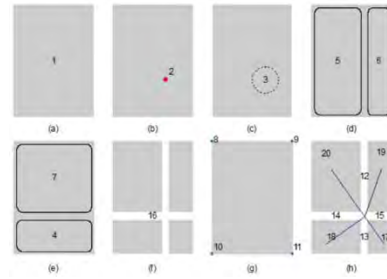
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A paper with or without a tiny dot

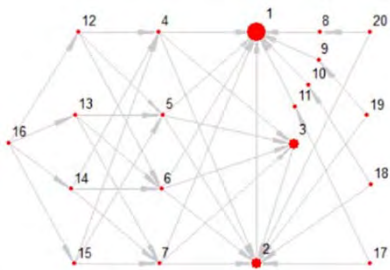A blank sheet of paper   Blank sheet with a single dot

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20 latent centers for a paper with a dot

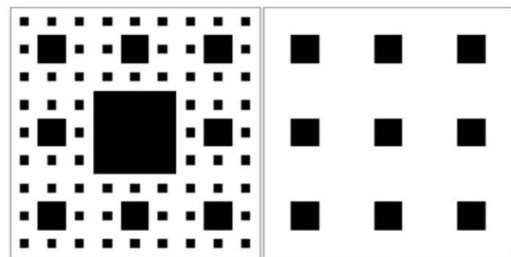Jiang B. (2016), A complex-network perspective on Alexander's wholeness, http://arxiv.org/abs/1602.08939

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A network perspective on the wholeness

Jiang B. (2016), A complex-network perspective on Alexander's wholeness, http://arxiv.org/abs/1602.08939

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Fractal versus Euclidean patterns

Jiang B. (2015), A city is a complex network, in: M. W. Mehaffy (editor, 2015), *Christopher Alexander A City is Not a Tree: 50th Anniversary Edition*, Sustasis Press: Portland, OR, 89-98.

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Conclusion – Part 1

- Alexander's life works as a whole for creating a new way of doing architecture, or.
- A new scientific underpinning for architecture and urban design.
- A new cosmology – a new conception of how the physical universe is put together, which is organic with the 'self' inside.
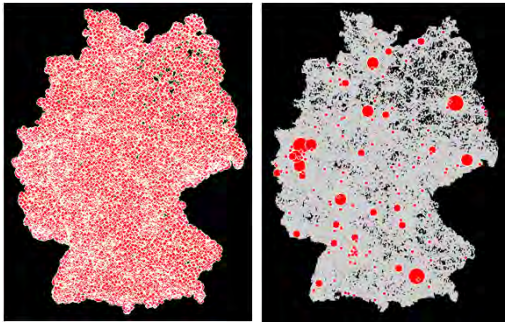- We must abandon Euclidean geometric thinking, and adopt instead fractal or living geometric thinking.

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Part 2
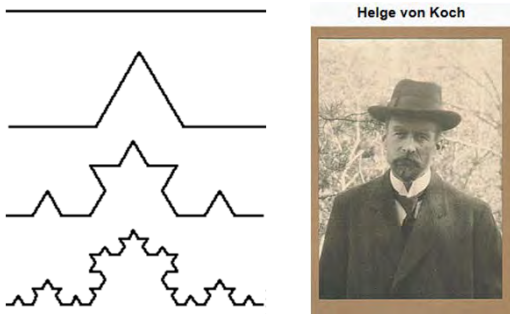Fractals or living structures emerged from big data
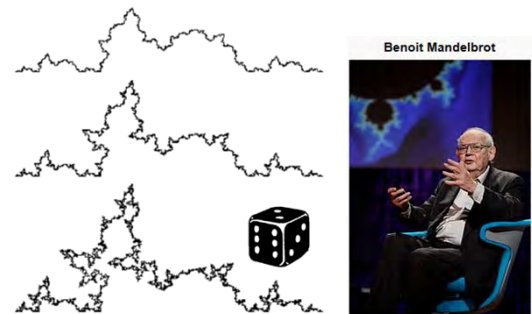

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Head/tail breaks

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Cantor set (1874)

Georg Cantor

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Koch curve (1904)

Helge von Koch

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Randomized or statistical Koch curves
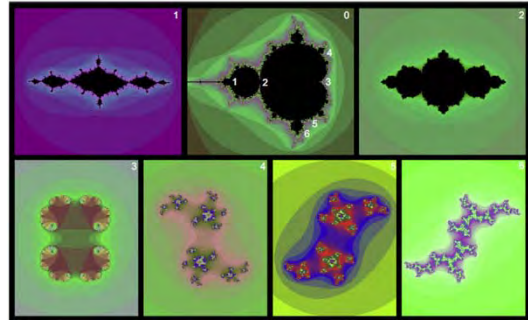

Benoit Mandelbrot

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Generative fractals – fractal sheep
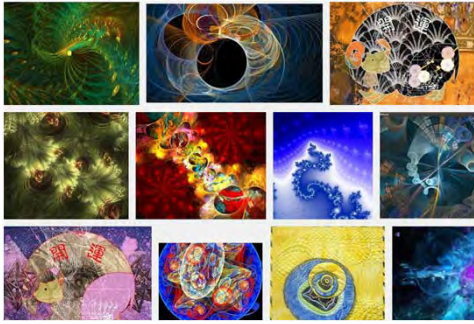
"Pretty pictures but pretty useless"

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Mandelbrot set – Julia sets ($z = z^2 + c$)

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Hidden order: Watts Towers
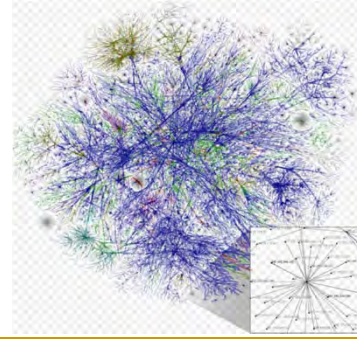

48

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Hidden order: Watts Towers (detailed looks)

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Internet and WWW

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Euclidean and fractal geometry in perspective (I)
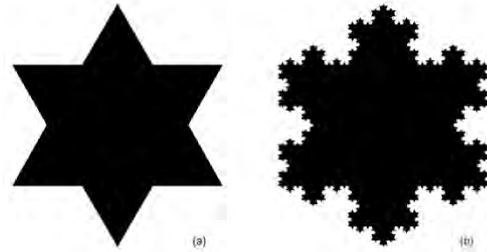

Regular + fraction
Euclidean and Gaussian: more or less similar things

Irregular + fractal
Fractal and Paretian: far more small things than large ones

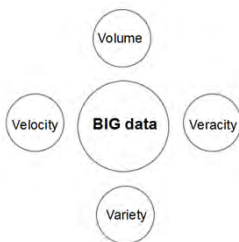51

Euclidean and fractal geometry in perspective (II)

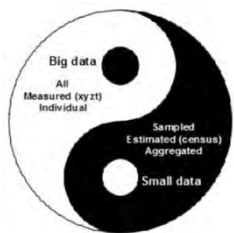(a)

(b)

Cold and dry – Benoit Mandelbrot

Living structue – Christopher Alexander

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What is BIG data?

Volume

Velocity **BIG data** Veracity

Variety

Big data
All Measured (xyzt) Individual

Sampled Estimated (census) Aggregated

Small data
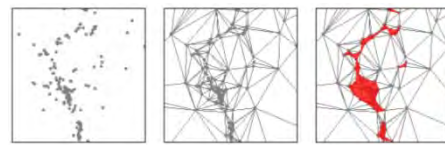
90% of world's data generated over last two years

Openness and transparency

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The notion of natural cities

- Natural cities refer to objectively or naturally defined and delineated human settlements, or human activities in general on the Earth's surface, using massive geographic information of various kinds, and based on head/tail breaks.
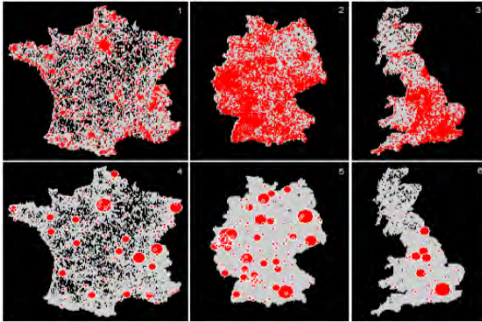

Points

Points + TIN

Points + TIN + natural cities
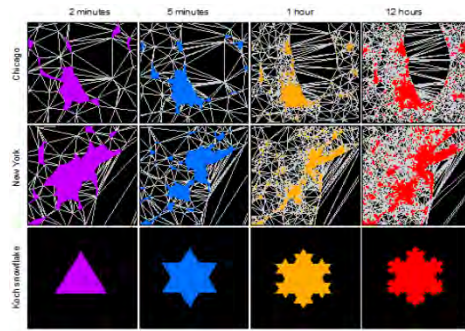
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Natural cities from POI
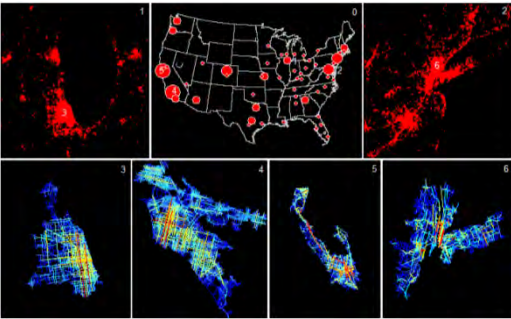

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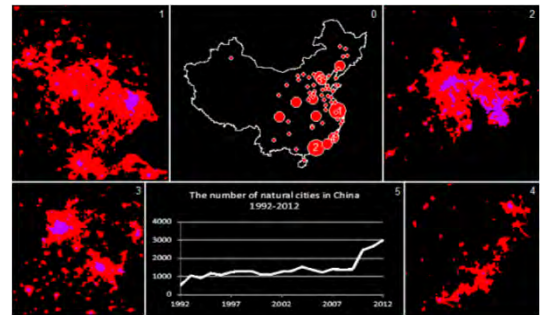## Natural cities from Tweets location data

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Scaling patterns at different levels
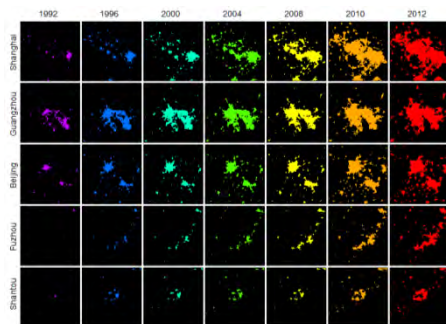

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China urbanization during 1992-2012

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Snapshots of the five largest regions

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Conclusion – Part 2

- Head/tail breaks leads to a new definition of fractal. A structure is fractal if scaling pattern of far more small things than large ones recurs multiple times.
- Head/tail breaks can be an efficient and effective visualization tool.
- Head/tail breaks is more natural than natural breaks.
- Scaling or fractal is more normal than normal distribution
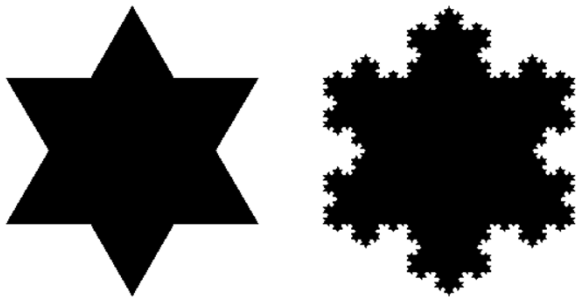
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Part 3
A mathematical model of beauty

A mathematical of beauty (beautimeter)

- A mathematical model of beauty or wholeness, which captures
- (1) not only Christopher Alexander's definition of wholeness, which is recursive, (2) but also human intuitions on beauty;
- for understanding (1) not only why a design is beautiful, (2) but also how much beauty the design has.

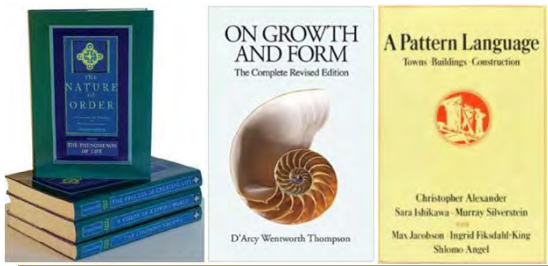---

Platonic solids

Cold and dry – Benoit Mandelbrot

Living structue – Christopher Alexander

The Nature of Order (30 years in the making)

- The order in nature and in what we make or build are essentially the same.

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or·der
/ˈôrdər/ 🔊

noun

1. the arrangement or disposition of people or things in relation to each other according to a particular sequence, pattern, or method.
   "I filed the cards in alphabetical order"
   synonyms: sequence, arrangement, organization, disposition, system, series, succession; More

2. an authoritative command, direction, or instruction.
   "he was not going to take orders from a mere administrator"
   synonyms: command, instruction, directive, direction, decree, edict, injunction, mandate, dictate, commandment, rescript; More

verb

1. give an authoritative direction or instruction to do something.
   "she ordered me to leave"
   synonyms: instruct, command, direct, enjoin, tell, require, charge; More

2. request (something) to be made, supplied, or served.
   "my friend ordered the tickets last week"
   synonyms: request, apply for, place an order for; More

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Many great thinkers in design

- Jane Jacobs - *The death and life of great American cities*
- Ian Mcharg – *Design with nature*
- Christopher Alexander – *A city is not a tree, A pattern language*
- After 30 years of dedicated work, Alexander came to a solution to good designs.
- What is good? Or how good is good?
- What is order? Or how order is order?

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Order vs disorder

- sequence, arrangement, organization, series, succession;
- grouping, classification, categorization, codification, systematization
- "alphabetical order"

- untidiness, disorderliness, mess, disarray, chaos, confusion;
- clutter, jumble;
- a muddle, a shambles
- "he hates disorder"

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- Zipf's law is an order

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To measure the degree of hidden order

- A mathematical model of wholeness is developed, and it is composed of two parts.
- A hierarchical graph in which nodes represent individual centers, and links point from small centers to related large centers.
- Degrees of beauty for the individual centers based on PageRank scores.
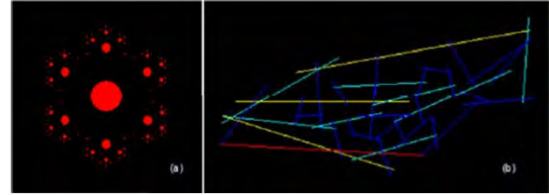- Degree of beauty for a whole derived from PageRank scores using head/tail breaks.

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Hierarchical graphs

(a)　(b)
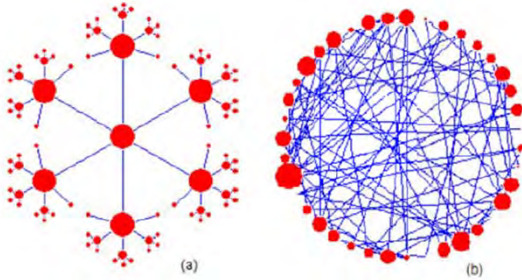
79

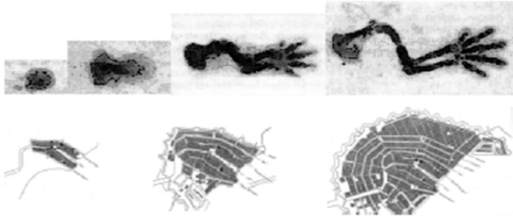## PageRank scores as degrees of beauty

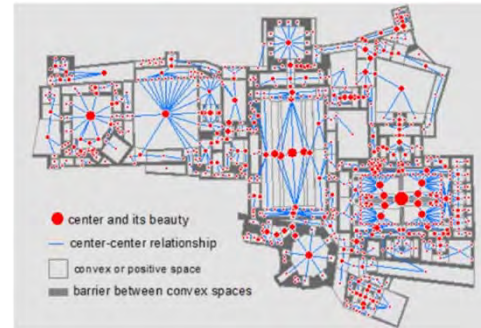$$r(i) = \frac{1-d}{n} + d \sum_{j \in ON(i)} \frac{r(j)}{n_j} \qquad [1]$$

(a)　(b)

80

Ht-index for degree of a whole

$$h = m(r) + 1 \qquad [2]$$

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Case study (Alhambra)

● center and its beauty
— center-center relationship
□ convex or positive space
▪ barrier between convex spaces

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Conclusion –Part 3

- Alexandrian order or living structure differs from regularity in Euclidean geometry or normality in Gaussian statistics
- We developed a mathematical model of wholeness or beauty to understand (1) not only why a design is beautiful, (2) but also how much beauty the design has.
- The case studies illustrate that the computed degrees of beautify capture fairly well (1) our intuition or the sense of beauty, and (2) Alexander's definition of wholeness.
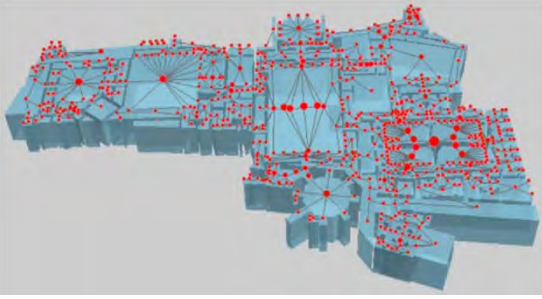
83

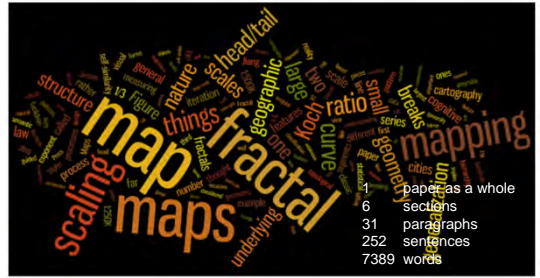Part 4
Sustainable urban design (adaptation + differentiation)

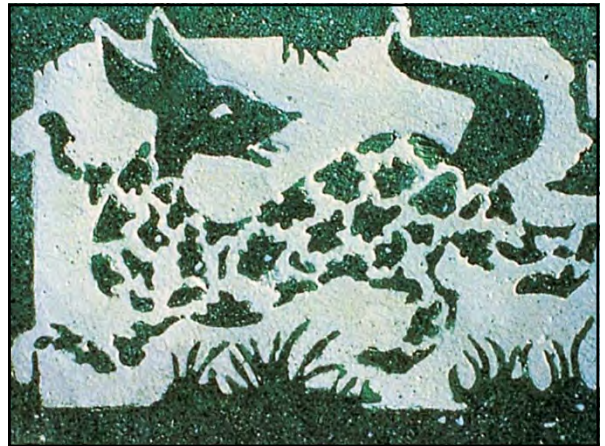
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The next step for sustainable urban design

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The timeless way of writing

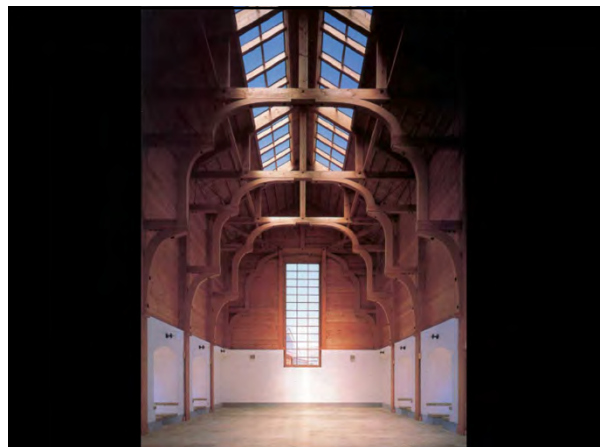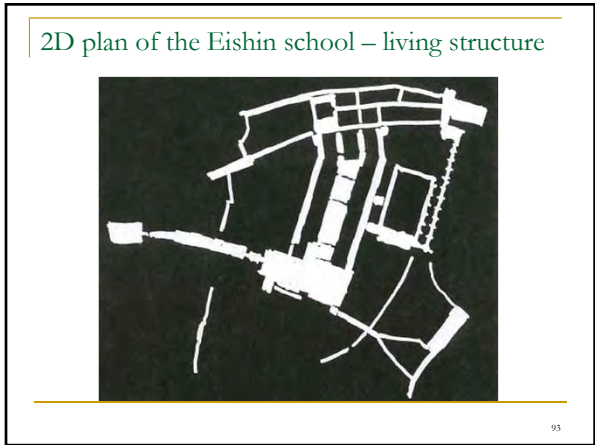| 1	paper as a whole
6	sections
31	paragraphs
252	sentences
7389	words

Jiang B. (2015), The fractal nature of maps and mapping, *International Journal of Geographical Information Science*, 29(1), 159-174.

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2D plan of the Eishin school – living structure

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Conclusion – Part 4

- Sustainable urban design must be achieved through unfolding rather than assembly
- Two basic unfolding processes or wholeness-extending processes: local adaptation and global differentiation
- Local adaptation implies more or less similar things, or autocorrelation (first law of geography)
- Global differentiation implies far more small things than large ones (scaling law).
- Two types of coherence: locally and globally

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Thank you very much!

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